

$\pm 4^{\circ}5$ C., or 0.8 per cent., and for *all* known melting points ± 15 , or 3 per cent. The calculated values for a large number of unknown melting and boiling points are given.

It is also shown that a knowledge of the melting or boiling points of the halogen compounds of an element may serve for the determination of the atomic weight when the application of the methods of specific heat and vapour density are inadmissible, or the results obtained by these contradictory.

An application of this is made in the case of beryllium, concerning the atomic weight of which there has recently been some dispute. From his determination of the specific heat of the metal, Emmerson Reynolds concludes that it is a dyad with atomic weight 9.2, whereas Nilson and Petterssen, from their determination of the specific heat, assign to it the atomic weight 13.8, in which case it would be trivalent. Now, according to calculation, the melting points of the chloride, bromide, and iodide of beryllium ought to be $(820-870)^{\circ}$ * $(802-820)^{\circ}$ and $(766-777)^{\circ}$ respectively. The boiling points, also, ought to *diminish* from the chloride to the iodide, if beryllium is a dyad with atomic weight 9.2. If it be a triad with atomic weight 13.8 these melting points ought to be about 500° lower, and the melting and boiling points ought to *increase* from the chloride to the iodide. With the object of settling this point the melting points of the chloride and bromide have been carefully determined, and it was found that the chloride melts at $858-890^{\circ}$, thus agreeing with the limits $(820-870)^{\circ}$ calculated for BeCl_2 ($\text{Be}=9.2$). The bromide also fuses between 858 and 890° , and at almost exactly the same temperature as the chloride, but if anything slightly higher, the calculated number for BeBr_2 being $802-820^{\circ}$. The rather high number found for the bromide is probably due to the substance being so readily volatile below its melting point that the heat absorbed during conversion into vapour cools the remainder of the solid, and thus prevents it melting so soon as it otherwise would do. These results confirm the view that beryllium is a dyad, with atomic weight 9.2; for though the melting point of the bromide is *apparently* slightly higher than that of the chloride, for the reason already given, yet BeBr_2 is far more volatile than BeCl_2 .

XIII. "Separate Enumerations of Primes of the Form $4n+1$ and of the Form $4n+3$." By J. W. L. GLAISHER, M.A., F.R.S., Fellow of Trinity College, Cambridge. Received June 11, 1879.

The present note contains the results of separate enumerations of primes of the forms $4n+1$ and $4n+3$ for seven groups each of 100,000

* Reckoned from absolute zero -273 .

numbers, viz., the subjoined tables give the number of primes of the form $4n+1$ and the number of primes of the form $4n+3$ in each ten thousand of the hundred thousand numbers of the first, second, third, fourth, seventh, eighth, and ninth millions. The properties of the $4n+1$ primes and the $4n+3$ primes are very different; and the idea of separately enumerating the two forms originated with Professor Tchebycheff, and was conveyed to me through M. Edouard Lucas.

The enumerations were effected as follows:—Each page of Burckhardt's and Dase's tables contains eighty lines corresponding to the eighty numbers 1, 7, . . . 97, 101, 103, . . . 199, 203, 209, . . . 299, *i.e.*, to the eighty numbers less than 300, and not divisible by 2, 3, or 5. The lines corresponding to numbers of the form $4n+1$ were ruled through in pencil, and then the bars (representing primes) which were ruled over were enumerated separately, and also the bars which were intact. The enumerations for the first three millions were made from Burckhardt's tables, and those for the seventh, eighth, and ninth millions from Dase's tables. In the case of the fourth million the enumerations were made from the proof sheets of my father's factor table of this million, the whole of which is now printed, and, with the exception of the few last pages, stereotyped.

The results for the groups in the first three millions were communicated to the Meeting of the British Association at Dublin, and are published on p. 471 of the Report for 1878. It is there stated that "the numbers given in the table are the result of a duplicate enumeration; but a third enumeration will be required, in order to render it certain that they are absolutely free from error." This third enumeration has now been made and the following two errors in the British Association table were detected by means of it: in the first ten thousand of the second million the numbers of $4n+1$ and $4n+3$ primes should be respectively 390 and 363, instead of 391 and 362 as printed, and in the third ten thousand of the third million the numbers should be 350 and 343 instead of 349 and 344 as printed. In the case of the other four millions, the results for which are here given for the first time, I have thought a careful duplicate calculation sufficient.

The number of $4n+1$ primes in each hundred was found by counting, and entered in its place on a printed form; and the number of $4n+3$ primes was similarly found and entered on another form. As a verification, the two results for each hundred were added together, and entered on a third form. These totals should agree with the numbers of primes in each hundred, which had been obtained in the previous enumeration that I had made of all the primes in the seven millions; and such was found to be the case. It is to be observed, however, that any error arising from the inaccurate or imperfect ruling of a pencil line would not be detected in this manner, as such an error would consist in a $4n+1$ prime being

counted as a $4n+3$ prime, or *vice versa*; but pains were taken to verify completely that the pencil lines had been ruled correctly, and for two of the millions (the seventh and ninth) the ruling was performed twice independently, in the one case a second copy of Dase being used, and in the other the original lines having been rubbed out before the second calculation was commenced. The enumerations were however troublesome, and it was much more difficult to ensure freedom from error than in the case of the simple enumerations of primes.

A specimen of a portion of one of the printed forms for the entry of the number of primes in each hundred, which have been referred to above, is given on p. 102 of vol. vii of the "Messenger of Mathematics"* (1877), and a preliminary account of the results of the complete enumeration of all the primes in the first, second, third, seventh, eighth, and ninth millions has been printed in the "Proceedings of the Cambridge Philosophical Society," vol. iii, pp. 17-23, 47-56 (1876-77).† The results of the enumeration for the fourth million will appear in the volume containing the factor table for that million.

The total numbers of primes of the forms $4n+1$ and $4n+3$ in the first one hundred thousand numbers of each million, taken from the subjoined tables, are

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
0— 100,000	4,784	4,808	-24	9,592
1,000,000—1,100,000	3,642	3,574	+68	7,216
2,000,000—2,100,000	3,463	3,411	+52	6,874
3,000,000—3,100,000	3,368	3,308	+60	6,676
6,000,000—6,100,000	3,193	3,204	-11	6,397
7,000,000—7,100,000	3,182	3,187	-5	6,369
8,000,000—8,100,000	3,126	3,124	+2	6,250

The most noticeable feature in these numbers is the excess of the $4n+1$ primes over the $4n+3$ primes in the groups taken from the second, third, and fourth millions; in these three groups there are 10,473 primes of the form $4n+1$, and 10,293 of the form $4n+3$, the difference being 180, and the total number of primes 20,766.

* "On Long Successions of Composite Numbers," vol. vii, pp. 102-106, 171-176.

† An enumeration of prime-pairs (*i.e.*, of pairs of primes separated by only one number), is given in the "Messenger of Mathematics," vol. viii, pp. 28-33 (1878); and a full account of Burekhardt's and Dase's tables occurs in a paper "On Factor Tables, with an account of the mode of formation of the Factor Table for the Fourth Million" ("Proceedings of the Cambridge Philosophical Society," vol. iii, pp. 99-138, 228-229, 1878). See also "British Association Report," 1878, pp. 172-178.

The results for the whole seven groups are

Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
24,758	24,616	142	49,374

It should be stated that 1 was counted as a prime of the form $4n+1$; 2, of course, was not counted at all.

TABLES showing the numbers of primes of the form $4n+1$ and of the form $4n+3$ in each ten thousand of the first hundred thousand numbers of the first, second, third, fourth, seventh, eighth, and ninth millions.

First Million.

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
0— 10,000...	610	619	— 9	1,229
10,000— 20,000...	516	517	— 1	1,033
20,000— 30,000...	486	497	— 11	983
30,000— 40,000...	474	484	— 10	958
40,000— 50,000...	464	466	— 2	930
50,000— 60,000...	469	455	+ 14	924
60,000— 70,000...	431	447	— 16	878
70,000— 80,000...	454	448	+ 6	902
80,000— 90,000...	445	431	+ 14	876
90,000— 100,000...	435	444	— 9	879
0— 100,000...	4,784	4,808	— 24	9,592

Second Million.

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
1,000,000—1,010,000...	390	363	+ 27	753
1,010,000—1,020,000...	367	352	+ 15	719
1,020,000—1,030,000...	369	363	+ 6	732
1,030,000—1,040,000...	347	354	— 7	701
1,040,000—1,050,000...	361	370	— 9	731
1,050,000—1,060,000...	349	349	0	698
1,060,000—1,070,000...	362	354	+ 8	716
1,070,000—1,080,000...	372	350	+ 22	722
1,080,000—1,090,000...	352	354	— 2	706
1,090,000—1,100,000...	373	365	+ 8	738
1,000,000—1,100,000...	3,642	3,574	+ 68	7,216

Third Million.

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
2,000,000—2,010,000....	351	354	— 3	705
2,010,000—2,020,000....	345	346	— 1	691
2,020,000—2,030,000....	350	343	+ 7	693
2,030,000—2,040,000....	340	350	—10	690
2,040,000—2,050,000....	343	328	+15	671
2,050,000—2,060,000....	356	340	+16	696
2,060,000—2,070,000....	355	339	+16	694
2,070,000—2,080,000....	343	331	+12	674
2,080,000—2,090,000....	339	347	— 8	686
2,090,000—2,100,000....	341	333	+ 8	674
2,000,000—2,100,000....	3,463	3,411	+52	6,874

Fourth Million.

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
3,000,000—3,010,000....	351	319	+32	670
3,010,000—3,020,000....	325	334	— 9	659
3,020,000—3,030,000....	322	341	—19	663
3,030,000—3,040,000....	322	335	—13	657
3,040,000—3,050,000....	354	317	+37	671
3,050,000—3,060,000....	341	316	+25	657
3,060,000—3,070,000....	334	330	+ 4	664
3,070,000—3,080,000....	357	338	+19	695
3,080,000—3,090,000....	339	347	— 8	686
3,090,000—3,100,000....	323	331	— 8	654
3,000,000—3,100,000....	3,368	3,308	+60	6,676

Seventh Million.

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
6,000,000—6,010,000....	341	320	+21	661
6,010,000—6,020,000....	317	321	— 4	638
6,020,000—6,030,000....	315	349	—34	664
6,030,000—6,040,000....	295	306	—11	601
6,040,000—6,050,000....	322	305	+17	627
6,050,000—6,060,000....	323	337	— 8	666
6,060,000—6,070,000....	322	293	+29	615
6,070,000—6,080,000....	326	327	— 1	653
6,080,000—6,090,000....	311	317	— 6	628
6,090,000—6,100,000....	315	329	—14	644
6,000,000—6,100,000....	3,193	3,204	—11	6,397

Eighth Million.

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
7,000,000—7,010,000....	320	309	+ 11	629
7,010,000—7,020,000....	306	324	— 18	630
7,020,000—7,030,000....	330	324	+ 6	654
7,030,000—7,040,000....	312	336	— 24	648
7,040,000—7,050,000....	327	330	— 3	657
7,050,000—7,060,000....	320	308	+ 12	628
7,060,000—7,070,000....	305	289	+ 16	594
7,070,000—7,080,000....	333	327	+ 6	660
7,080,000—7,090,000....	324	316	+ 8	640
7,090,000—7,100,000....	305	324	— 19	629
7,000,000—7,100,000....	3,182	3,187	— 5	6,369

Ninth Million.

	Number of $4n+1$ primes.	Number of $4n+3$ primes.	Difference.	Total number of primes.
8,000,000—8,010,000....	304	333	— 29	637
8,010,000—8,020,000....	295	314	— 19	609
8,020,000—8,030,000....	323	308	+ 15	631
8,030,000—8,040,000....	307	298	+ 9	605
8,040,000—8,050,000....	328	311	+ 17	639
8,050,000—8,060,000....	331	321	+ 10	652
8,060,000—8,070,000....	316	312	+ 4	628
8,070,000—8,080,000....	308	315	— 7	623
8,080,000—8,090,000....	323	296	+ 27	619
8,090,000—8,100,000....	291	316	— 25	607
8,000,000—8,100,000....	3,126	3,124	+ 2	6,250

XIV. "On the Effects of Chloroform and Ether on the Radial Tracing." By C. HANDFIELD JONES, M.B. Cantab, F.R.S.
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(Abstract.)

The effects of chloroform and ether on the pulse, when employed to produce anæsthesia in surgical operations, vary considerably. Sometimes the heart's force, estimated by the lowering of the primary rise, with due attention to the spring pressure, is greatly diminished, sometimes it is increased. The rate of the pulse is generally increased. The rhythm is often deranged, sometimes very much, and so-called